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Volatility Research Notes

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What is Volatility?

When the price of a security experiences price fluctuations that are unpredictable or have sudden, sharp price movements, it can be described as volatile. In contrast, a low volatility security would have a more stable, predictable price[d]. A common measure of volatility for individual stocks is the stock's beta. This number compares the movements of an individual security against those of a benchmark index[e]. It can also be measured with the standard deviation of a market or security's annualized returns over a given period. Maximum drawdown, or the largest historical loss, is another way to measure stock price volatility, and it is used by speculators, asset allocators, and growth investors to limit their losses[f].

Sharpe Ratio[d]

The Sharpe ratio is used for measuring the performance of an investment by comparing the average return in relation to its volatility, helping you gauge risk. Given by $\varphi = \frac{(\mu - \Gamma)}{\sigma}$, where μ is the average return, Γ is the risk-free interest rate, and σ is the standard deviation of returns.

Volatility is Inherently Latent and Stochastic[a]

Volatility is inherently latent and evolves stochastically over time. Volatility is uncertain, and so is the level of uncertainty. This means that it is impossible to fully predict volatility and true volatility cannot be determined, only found with error. There is only a limited set of observations available, even in extremely liquid markets. If not all information that goes into volatility can be forecasted, then it can be assumed that we can only use a subset of all information.

Integrated Variance and Spot Variance[a]

The volatility for the continuous-time process over the interval $[t - 1, t]$ is related to spot volatility, $\sigma(s)$. There are many models for calculating spot volatility, which will be touched on later.
We can measure integrated variance over a time interval as integrated variance, or

$$IV(t) = \int_{t}^{t-1} \sigma^2(s)ds.$$

Integrated variance is particularly important in the realization of variance (RV), which is an empirical estimate of integrated variance obtained from high-frequency intraday data. Realized variance is calculated by summing up squared returns over a specified period, usually one day, based on intraday data (e.g., minute-by-minute or second-by-second returns). This provides a non-parametric measure of the total variance over the period and is used in various applications, including volatility forecasting, risk management, and in the calibration of continuous-time models and volatility models like GARCH. It measures the total expected variance over a period.

GARCH[c]

GARCH, or generalized autoregressive conditional heteroskedasticity, takes on several forms. Known for its effectiveness in modeling asset returns and inflation, it is not very effective when it comes to the occurrence of black swan events.

It is used to estimate the return volatility of stocks, bonds, and other investments by providing real-world context alongside other factors. The GARCH model has 3 main steps: estimate a best-fitting autoregressive model, compute autocorrelations of the error term, and then test for significance. GARCH processes are autoregressive, so they depend on past observations and variances.

The basic GARCH model is defined as so:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \text{ or}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where ω is the long-run average variance, α represents the impact of past squared residuals (or shocks) on the current period's variance, and β is a measure of the impact of past conditional variances on the current period's variance, and ε_{t-1} is the lagged error term or residual.

Glosten-Jagannathan-Runkle GARCH (GJR-GARCH)[g][h][j]

The GJR-GARCH adds on an additional layer, similar to a machine learning model’s activation layer, to the model in order to better capture the "leverage effect", where negative asset returns may increase future volatility more than positive returns of the same magnitude[i].

It can be written as so:

$$\sigma_t^2 = \omega + (\alpha + \gamma_i I_{\varepsilon_{t-1}<0})\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \text{ or}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i + \gamma_i I_{t-i})\varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

Where $I_{\varepsilon_{t-1}<0}$ is the indicator/activation function, and γ captures the additional impact of negative shocks on current volatility. Specifically, γ measures equally sized negative shock ε_{t-i} (compared to a positive shock) contributes to the increase in volatility.

Stochastic Volatility[k][l]

Since we have established that volatility is inherently latent and stochastic, we can attempt to look at stochastic methods of modeling volatility. The simplest version of a stochastic volatility model is given by

$$y_t = e^{(\frac{h_t}{2})} \times u_t$$

$$h_t = \mu + \phi(h_{t-1} - \mu) + \sigma\eta_t, t \le n$$

where y_t is the response variable (the log returns of a stock price), h_t is the standard deviation of the returns of y_t on a log scale, and the errors u_t and η_t are Gaussian white noise sequences. It suggests that the current log-volatility h_t is a function of its previous value h_{t-1} , and a combination of parameters ϕ

and σ that controls the level and persistence of volatility. We can extend this to Markov chain Monte Carlo methods (MCMC) to obtain a simulated sequence of random volatility samples.

My Proposal Idea Based on Kaggle Volatility Competition, and the NN/Statistical Approach[m][n][o][p]

Using orderbook data, we can identify moments of low liquidity (i.e. a bid-ask spread that is not being met), and then use that to predict increased volatility in the short term. Liquidity and volatility are inversely related, so finding those moments can help us trade volatility. The statistical approach would be to have a parametric combination of factors, similar to a GARCH process, and forecasting volatility using that. The GJR-GARCH model is similar to how a neural network is designed, so machine learning techniques could be implemented to create a deeper net for time-series prediction. By using stochastic models, we could also create a risk profile or create bounds of volatility that we can abide by to minimize risk.

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